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BRST Symmetric Formulation of a Theory with Gribov-type Copies

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Abstract

A path integral with BRST symmetry can be formulated by summing the Gribov-type copies in a very specific way if the functional correspondence between τ and the gauge parameter ω defined by $\tau(x) = f(A_\mu^\omega)$ is "globally single valued", where $f(A_\mu^\omega) = 0$ specifies the gauge condition. A soluble gauge model with Gribov-type copies recently analyzed by Friedberg, Lee, Pang and Ren satisfies this criterion. A detailed BRST analysis of the soluble model proposed by the above authors is presented. The BRST symmetry, if it is consistently implemented, ensures the gauge independence of physical quantities. In particular, the vacuum (ground) state and the perturbative corrections to the ground state energy in the above model are analysed from a view point of BRST symmetry and R_ξ-gauge. Implications of the present analysis on some aspects of the Gribov problem in non-Abelian gauge theory, such as the $1/N$ expansion in QCD and also the dynamical instability of BRST symmetry, are briefly discussed.

1 Introduction

The quantization of non-Abelian gauge theory is complicated by the presence of the so-called Gribov problem[1][2][3][4]. The Gribov problem in general suggests that the Coulomb gauge cannot completely fix the gauge due to the presence of more than one gauge configurations which satisfy the Coulomb gauge condition or , in certain circumstances in compactified space-time, it even suggests the absence of the gauge configuration which satisfies the Coulomb gauge condition [2]. In Euclidean formulation of gauge theory, the Landau gauge also suffers from the same complications. Although the full details of the Gribov problem are not understood yet, a working prescription is figured out if one regards the Gribov problem as the appearance of several gauge copies(i.e., if one assumes that one can always find at least one gauge configuration which satisfies the Coulomb gauge condition). In fact, it was noted some time ago[5] that if the functional correspondence between τ and the gauge parameter ω defined by

$$\tau(x) = \partial^{\mu} A_{\mu}^{\omega}(x) \tag{1.1}$$

is "globally single valued", the path integral with BRST symmetry[6] is defined by summing over Gribov copies in a very specific way. For a more general gauge fixing, (1.1) may be replaced by

$$\tau(x) = f(A_{\mu}^{\omega}(x)) \tag{1.2}$$

where $f(A_{\mu}^{\omega}(x)) = 0$ specifies the gauge condition. In (1.1) or (1.2) $A_{\mu}^{\omega}(x)$ stands for the gauge field obtained from $A_{\mu}(x)$ by a gauge transformation specified by the gauge orbit parameter $\omega(x)$. For an infinitesimal $\omega(x)$, one has

$$A_{\mu}^{a\omega}(x) = A_{\mu}^{a}(x) + \partial_{\mu}\omega^{a}(x) - g f^{abc} A_{\mu}^{b}(x)\omega^{c}(x) \tag{1.3}$$

One may regard (1.1) or (1.2) as a functional correspondence between τ and ω parametrized by A_{μ}. The globally single valued correspondence between τ and ω, which is explained in more detail in Section 2, is required for any value of A_{μ} to write a simple path integral formula. [If one restricts oneself to only one of the Gribov copies by some means[7], one can also incorporate the idea of BRST symmetry. But a simple prescription which selectively picks up only one copy appears to be misssing at this moment.]

Recently a very detailed analysis of the Gribov problem was performed by Friedberg, Lee ,Pang and Ren[8] on the basis of a soluble gauge model which exhibits Gribov-type copies. One of the main conclusions in [8] is that the singularity associated with the so-called Gribov horizon is immaterial at least in their soluble model and one may incorporate all the Gribov copies in a very specific way. They showed how this prescription works in the soluble model proposed by them.

The present work is motivated by the fact that the model and the gauge choice in Ref.[8] satisfy our criterion in (1.1) or (1.2). One can thus formulate a BRST invariant path integral for the model proposed in [8] by summing over Gribov-type copies ; the physical quantities thus calculated agree with those in Ref.[8]. Note that the BRST symmetry deals with an extended Hilbert space which contains indefinite metric in general, although the physical sector specified by BRST cohomology contains only positive metric. Since the BRST symmetry plays a fundamental role in modern gauge theory, we here present a detailed BRST analysis of the soluble model proposed in [8].

2 BRST invariant path integral in the presence of Gribov copies

In this Section we recapitulate the essence of the argument presented in [5]. We start with the Faddeev-Popov formulation of the Feynman-type gauge condition [9][10]. The vacuum-to-vacuum transition amplitude is defined by

$$\langle +\infty | -\infty \rangle = \int \mathcal{D}A_\mu^\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C)\Delta(A) exp\{iS(A_\mu^\omega) - \frac{i}{2\alpha}\int C(x)^2 dx\} \tag{2.1}$$

where $S(A_\mu^\omega)$ stands for the action invariant under the Yang-Mills local gauge transformation. The positive constant α is a gauge fixing parameter which specifies the Feynman-type gauge condition. [The equations in this Section are written in the Minkowski metric, but they should really be interpreted in the Euclidean metric to render the functional integral and the Gribov problem well-defined.] In the following we often suppress the internal symmetry indices, and instead we write the gauge parameter explicitly: A_μ^ω indicates the gauge field which is obtained from A_μ by a gauge transformation specified by $\omega(x)$. The

determinant factor $\Delta(A)$ is defined by[9]

$$\begin{aligned}\Delta(A)^{-1} &= \int \mathcal{D}\omega\mathcal{D}C\ \delta(\partial^{\mu}A_{\mu}^{\omega} - C)exp\{-\frac{i}{2\alpha}\int C(x)^2dx\} \\ &\approx const\{\sum_k |det[\frac{\partial}{\partial\omega_k}\partial^{\mu}A_{\mu}^{\omega_k}]|^{-1}\}\end{aligned} \tag{2.2}$$

where the summation runs over all the gauge equivalent configurations satisfying $\partial^{\mu}A_{\mu}^{\omega_k} = 0$, which were found by Gribov[1] and others[2][3][4]. Equation(2.2) is valid only for sufficiently small α, since the parameter $\omega'(\omega, A, C)$ defined by

$$\partial^{\mu}A_{\mu}^{\omega'(\omega,A,C)} = \partial^{\mu}A_{\mu}^{\omega} - C \tag{2.3}$$

has a complicated branch structure for large C in the presence of Gribov ambiguities. Obviously the Feynman-type gauge formulation becomes even more involved than the Landau-type gauge condition.

It was suggested in [5] to replace equation (2.1) by

$$\langle+\infty|-\infty\rangle = \frac{1}{N}\int\mathcal{D}A_{\mu}^{\omega}\mathcal{D}C\delta(\partial^{\mu}A_{\mu}^{\omega} - C)det[\frac{\partial}{\partial\omega}\partial^{\mu}A_{\mu}^{\omega}]exp\{iS(A_{\mu}^{\omega}) - \frac{i}{2\alpha}\int C(x)^2dx\} \tag{2.4}$$

The crucial difference between (2.1) and (2.4) is that (2.4) is local in the gauge space $\omega(x)$ (i.e., the gauge fixing factor and the compensating factor are defined at the identical ω),whereas $\Delta(A)$ in (2.1) is gauge independent and involves a non-local factor in ω as is shown in (2.2). As the determinant in (2.4) depends on A_{μ}^{ω}, the entire integrand in (2.4) is in general no more degenerate with respect to gauge equivalent configurations even if the gauge fixing term itself may be degenerate for certain configurations. Another important point is that one takes the absolute values of determinant factors in (2.2) thanks to the definition of the δ-function, whereas just the determinant which can be negative as well as positive appears in (2.4). It is easy to see that (2.4) can be rewritten as

$$\langle+\infty|-\infty\rangle = \frac{1}{\tilde{N}}\int\mathcal{D}A_{\mu}^{\omega}\mathcal{D}B\mathcal{D}\bar{c}\mathcal{D}c\ exp\{iS(A_{\mu}^{\omega}) + i\int\mathcal{L}_g dx\} \tag{2.5}$$

where

$$\mathcal{L}_g = -\partial^{\mu}B^aA_{\mu}^{a\omega} + i\partial^{\mu}\bar{c}^a(\partial_{\mu} - gf^{abc}A_{\mu}^{b\omega})c^c + \frac{\alpha}{2}B^aB^a \tag{2.6}$$

with B^a the Lagrangian multiplier field, and $\bar{c}^a$ and c^a the (hermitian) Faddeev-Popov ghost fields; f^{abc} is the structure constant of the gauge group and g is the gauge coupling

constant. If one imposes the hermiticity of $\bar{c}^a$ and c^a, the phase factor of the determinant in (2.4) cannot be removed. The normalization constant $\tilde{N}$ in (2.5) includes the effect of Gaussian integral over B in addition to N in (2.4), and in fact $\tilde{N}$ is independent of α. See eq. (2.10).

This $\mathcal{L}_g$ as well as the starting gauge invariant Lagrangian are invariant under the BRST transformation defined by

$$\begin{aligned}
\delta_\theta A_\mu^{a\omega} &= i\theta[\partial_\mu c^a - g f^{abc} A_\mu^{b\omega} c^c] \\
\delta_\theta c^a &= i\theta(g/2) f^{abc} c^b c^c \\
\delta_\theta \bar{c}^a &= \theta B^a \\
\delta_\theta B^a &= 0
\end{aligned} \tag{2.7}$$

where θ and the ghost variables $c^a(x)$ and $\bar{c}^a(x)$ are elements of the Grassmann algebra , i.e., $\theta^2 = 0$. This transformation can be confirmed to be nil-potent $\delta^2 = 0$, for example,

$$\delta_{\theta_2}(\delta_{\theta_1} A_\mu^{a\omega}) = 0 \tag{2.8}$$

One can also confirm that the path integral measure in (2.5) is invariant under (2.7). Note that the transformation (2.7) is "local" in the ω parameter; precisely for this property, the prescription in (2.4) was chosen.

To interprete the path integral measure in (2.4) as the path integral over all the gauge field configurations divided by the gauge volume, namely

$$\langle +\infty | -\infty \rangle = \int \frac{\mathcal{D}A_\mu^\omega}{gauge\ volume(\omega)}\ exp\{iS(A_\mu^\omega)\} \tag{2.9}$$

one needs to define the normalization factor in (2.4) by

$$\begin{aligned}
N &= \int \mathcal{D}\omega \mathcal{D}C \delta(\partial^\mu A_\mu^\omega - C) det[\frac{\partial}{\partial\omega}\partial^\mu A_\mu^\omega] exp\{-\frac{i}{2\alpha}\int C(x)^2 dx\} \\
&= \int \mathcal{D}\omega det[\frac{\partial}{\partial\omega}\partial^\mu A_\mu^\omega] exp\{-\frac{i}{2\alpha}\int (\partial^\mu A_\mu^\omega)^2 dx\} \\
&= \int \mathcal{D}\tau\ exp\{-\frac{i}{2\alpha}\int \tau(x)^2 dx\}
\end{aligned} \tag{2.10}$$

where the function τ is defined by

$$\tau(x) \equiv \partial^\mu A_\mu^\omega(x) \tag{2.11}$$

and the determinant factor is regarded as a Jacobian for the change of variables from $\omega(x)$ to $\tau(x)$. Although we use the Feynman-type gauge fixing (2.11) as a typical example in this Section, one may replace (2.11) by

$$\tau(x) \equiv f(A_\mu^\omega(x)) \tag{2.12}$$

to deal with a more general gauge condition

$$f(A_\mu^\omega(x)) = 0 \tag{2.13}$$

It is crucial to establish that the normalization factor in (2.10) is independent of A_μ. Only in this case, (2.4) defines an acceptable vacuum transition amplitude. The Gribov ambiguity in the present case appears as a non-unique correspondence between $\tau(x)$ and $\omega(x)$ in (2.11), as is schematically shown in Fig. 1 which includes 3 Gribov copies. The path integral in (2.10) is performed along the contour in Fig. 1. As the Gaussian function is regular at any finite point, the complicated contour in Fig. 1 gives rise to the same result in (2.10) as a contour corresponding to $A_\mu = 0$. In the present path integral formulation, the evaluation of the normalization factor in (2.10) is the only place where we explicitly encounter the multiple solutions of gauge fixing condition.[If the normalization factor N should depend on gauge field A_μ, the factor N, which is gauge independent in the sense that we integrated over entire gauge orbit, needs to be taken inside the path integral in (2.4). In this case one looses the simplicity of the formula (2.4).]

The basic assumption we have to make is therefore that (2.11) in the context of the path integral (2.10) is "globally single-valued", in the sense that the asymptotic functional correspondence between ω and τ is little affected by a fixed A_μ with $\partial^\mu A_\mu = 0$[5]; Fig.1 satisfies this requirement. This assumption appears to be physically reasonable if the second derivative term of the gauge orbit parameter dominates the functional correspondence in (2.11), though it has not been established mathematically. To define the functional correspondence between ω and τ in (2.11), one needs in general some notion of norm such as L^2-norm for which the Coulomb gauge vacuum is unique [3][4]. The functional configurations which are square integrable however have zero measure in the path integral[11], and this makes the precise analysis of (2.11) very complicated: At least what we need to do is to start with an expansion of a generic field variable in terms of

some complete orthonormal basis set (which means that the field is inside the L^2-space) and then let each expansion coefficient vary from $-\infty$ to $+\infty$(which means that the field is outside the L^2-space).

If the Gribov problem simply means the situation as is schematically shown in Fig. 1, the prescription in (2.4) may be justified. The indefinite signature of the determinant factor in (2.4) is not a difficulty in the framework of indefinite metric field theory[12][13] since the determinant factor is associated with the Faddeev-Popov ghost fields and the BRST cohomology selects the positive definite physical space. On the other hand , the Gribov problem may aslo suggest that one cannot bring the relation(2.11) with fixed A_μ to $\partial^\mu A_\mu^\omega = 0$ by any gauge transformation[2]. If this is the case, the asymptotic behavior of the mapping (2.11) is in general modified by A_μ and our prescription cannot be justified. Consequently, the prescription (2.4) may be valid to the extent that one can always achieve the condition $\partial^\mu A_\mu = 0$ by means of suitable (but not necessarily unique) gauge transformations. We also note that the limit $\alpha \rightarrow large$ in (2.5) and (2.10) corresponds to a very loose gauge fixing.

3 BRST analysis of a soluble gauge model

3.1, A SOLUBLE GAUGE MODEL

The soluble gauge model of Friedberg, Lee, Pang and Ren[8] is defined by

$$\mathcal{L} = \frac{1}{2}\{[\dot{X}(t)+g\xi(t)Y(t)]^2+[\dot{Y}(t)-g\xi(t)X(t)]^2+[\dot{Z}(t)-\xi(t)]^2\}-U(X(t)^2+Y(t)^2) \quad (3.1)$$

where $\dot{X}(t)$, for example, means the time derivative of $X(t)$, and the potential U depends only on the combination X^2+Y^2. This Lagrangian is invariant under a local gauge transformation parametrized by $\omega(t)$,

$$\begin{aligned}
X^\omega(t) &= X(t)\cos g\omega(t) - Y(t)\sin g\omega(t) \\
Y^\omega(t) &= X(t)\sin g\omega(t) + Y(t)\cos g\omega(t) \\
Z^\omega(t) &= Z(t) + \omega(t) \\
\xi^\omega(t) &= \xi(t) + \dot{\omega}(t) \qquad (3.2)
\end{aligned}$$

The gauge condition (an analogue of $A_0 = 0$ gauge)

$$\xi(t) = 0 \tag{3.3}$$

or (an analogue of $A_3 = 0$ gauge)

$$Z(t) = 0 \tag{3.4}$$

is well-defined without suffering from Gribov-type copies. However, it was shown in [8] that the gauge condition (an analogue of the Coulomb gauge)

$$Z(t) - \lambda X(t) = 0 \tag{3.5}$$

with a constant λ suffers from the Gribov- type complications. This is seen by using the notation in (3.2) as

$$\begin{aligned} Z^{\omega}(t) - \lambda X^{\omega}(t) &= Z(t) + \omega(t) - \lambda X(t)\cos g\omega(t) + \lambda Y(t)\sin g\omega(t) \\ &= \omega(t) + \lambda\sqrt{X^2+Y^2}[\cos\phi(t) - \cos(g\omega(t) + \phi(t))] = 0 \end{aligned} \tag{3.6}$$

where we used the relation(3.5) and

$$X(t) = \sqrt{X^2+Y^2}\cos\phi(t), \quad Y(t) = \sqrt{X^2+Y^2}\sin\phi(t) \tag{3.7}$$

From a view point of gauge fixing, $\omega(t) = 0$ is a solution of (3.6) if (3.5) is satisfied. By analyzing the crossing points of two graphs in (ω, η) plane defined by

$$\begin{aligned} \eta &= \frac{1}{\lambda\sqrt{X^2+Y^2}}\omega \\ \eta &= \cos(g\omega + \phi) - \cos\phi \end{aligned} \tag{3.8}$$

one can confirm that eq.(3.6) in general has more than one solutions for ω.

From a view point of general gauge fixing procedure, we here regard the algebraic gauge fixing such as (3.3) and (3.4) well-defined; in the analysis of the Gribov problem in Ref.[2], the algebraic gauge fixing [3] is excluded.

The authors in Ref.[8] started with the Hamiltonian formulated in terms of the well-defined gauge $\xi(t) = 0$ in (3.3) and then faithfully rewrote the Hamiltonian in terms of the variables defined by the "Coulomb gauge" in (3.5). By this way, the authors in [8] analyzed in detail the problem related to the Gribov copies and the so-called Gribov horizons where

the Faddeev-Popov determinant vanishes. They thus arrived at a prescription which sums over all the Gribov-type copies in a very specific way. As is clear from their derivation, their specification satisfies the unitarity and gauge independence.

In the context of BRST invariant path integral discussed in Section 2, the crucial relation (2.11) becomes

$$\begin{aligned} \tau(t) &= Z^{\omega}(t) - \lambda X^{\omega}(t) \\ &= \omega(t) + Z(t) - \lambda X(t)\cos g\omega(t) + \lambda Y(t)\sin g\omega(t) \end{aligned} \tag{3.9}$$

in the present model. For $X = Y = 0$, the functional correspondence between ω and τ is one-to-one and monotonous for any fixed value of t . When one varies $X(t), Y(t)$ and $Z(t)$ continuously, one deforms this monotonous curve continuously. But the asymptotic correspondence between $\omega(t)$ and $\tau(t)$ at $\omega(t) = \pm\infty$ for each value of t is still kept preserved, at least for any fixed $X(t), Y(t)$ and $Z(t)$. This correspondence between $\omega(t)$ and $\tau(t)$ thus satisfies our criterion discussed in connection with (2.11). The absence of terms which contain the derivatives of $\omega(t)$ in (3.9) makes the functional correspondence in (3.9) well-defined and transparent.

From a view point of gauge fixing in (3.6), this "globally single-valued" correspondence between ω and τ means that one always obtains an *odd* number of solutions for (3.6). The prescription in [8] is then viewed as a sum of all these solutions with signature factors specified by the signature of the Faddeev-Popov determinant

$$det\{\frac{\partial}{\partial\omega(t')}[Z^{\omega}(t) - \lambda X^{\omega}(t)]\} = det\{[1 + \lambda g Y^{\omega}(t)]\delta(t - t')\} \tag{3.10}$$

evaluated at the point of solutions , $\omega = \omega(\sqrt{X^2 + Y^2}, \phi)$, of (3.6). The row and column indices of the matrix in (3.10) are specified by t and t', respectively. In the context of BRST invariant formulation, a pair-wise cancellation of Gribov-type copies takes place, except for one solution, in the calculation of the normalization factor in (2.10) or (3.21) below.

3.2, BRST INVARIANT PATH INTEGRAL

The relation (3.9) satisfies our criterion discussed in connection with (2.11). We can thus define an analogue of (2.5) for the Lagrangian (3.1) by

$$\langle +\infty | -\infty \rangle = \frac{1}{\tilde{N}} \int d\mu\ exp\{iS(X^{\omega}, Y^{\omega}, Z^{\omega}, \xi^{\omega}) + i\int \mathcal{L}_g dt\} \tag{3.11}$$

where

$$\begin{aligned} S(X^\omega, Y^\omega, Z^\omega, \xi^\omega) &= \int \mathcal{L}(X^\omega, Y^\omega, Z^\omega, \xi^\omega) dt \\ &= S(X, Y, Z, \xi) \end{aligned} \tag{3.12}$$

in terms of the Lagrangian $\mathcal{L}$ in (3.1). The gauge fixing part of (3.11) is defined by

$$\mathcal{L}_g = -\beta \dot{B} \xi^\omega + B(Z^\omega - \lambda X^\omega) + \beta i \dot{\bar{c}} \dot{c} - i\bar{c}(1 + g\lambda Y^\omega)c + \frac{\alpha}{2} B^2 \tag{3.13}$$

where α, β and λ are numerical constants, and $\bar{c}$ and c are (hermitian) Faddeev-Popov ghost fields. B is a Lagrangian multiplier field. Note that $\mathcal{L}_g$ is hermitian. The integral measure in (3.11) is given by

$$d\mu = \mathcal{D}X^\omega \mathcal{D}Y^\omega \mathcal{D}Z^\omega \mathcal{D}\xi^\omega \mathcal{D}B \mathcal{D}\bar{c} \mathcal{D}c \tag{3.14}$$

The Lagrangians $\mathcal{L}$ and $\mathcal{L}_g$ and the path integral measure (3.14) are invariant under the BRST transformation defined by

$$\begin{aligned} X^\omega(t,\theta) &= X^\omega(t) - i\theta g c(t) Y^\omega(t) \\ Y^\omega(t,\theta) &= Y^\omega(t) + i\theta g c(t) X^\omega(t) \\ Z^\omega(t,\theta) &= Z^\omega(t) + i\theta c(t) \\ \xi^\omega(t,\theta) &= \xi^\omega(t) + i\theta \dot{c}(t) \\ c(t,\theta) &= c(t) \\ \bar{c}(t,\theta) &= \bar{c}(t) + \theta B(t) \end{aligned} \tag{3.15}$$

where the parameter θ is a Grassmann number, $\theta^2 = 0$. Note that θ and ghost variables anti-commute. In (3.15) we used a BRST superfield notation: In this notation, the second component of a superfield proportional to θ stands for the BRST transformed field of the first component. The second component is invariant under BRST transformation which ensures the nil-potency of the BRST charge. In the operator notation to be defined later, one can write , for example,

$$X^\omega(t,\theta) = e^{-\theta Q} X^\omega(t,0) e^{\theta Q} \tag{3.16}$$

with a nil-potent BRST charge Q, $\{Q, Q\}_+ = 0$. Namely, the BRST transformation is a translation in θ-space, and θQ is analogous to momentum operator.

In (3.11)$\sim$ (3.15), we explicitly wrote the gauge parameter ω to emphasize that BRST transformation is "local" in the ω-space. For $\mathcal{L}_g$ in (3.13), the relation (3.9) is replaced by (an analogue of the Landau gauge)

$$\begin{aligned}\tau(t) &\equiv \beta\dot{\xi}^{\omega}(t) + Z^{\omega}(t) - \lambda X^{\omega}(t) \\ &= \beta\ddot{\omega}(t) + \omega(t) + \beta\dot{\xi}(t) + Z(t) - \lambda X(t)\cos g\omega(t) + \lambda Y(t)\sin g\omega(t)\end{aligned} \quad (3.17)$$

The functional correspondence between ω and τ is monotonous and one-to-one for weak $\xi(t), Z(t), X(t)$ and $Y(t)$ fields; this is understood if one rewrites the relation (3.17) for Euclidean time $t = -it_E$ by neglecting weak fields as

$$\tau(t_E) = (-\beta\frac{d^2}{dt_E^2} + 1)\omega(t_E)$$

The Fourier transform of this relation gives a one-to-one monotonous correspondence between the Fourier coefficients of τ and ω for non-negative β. The asymptotic functional correspondence between ω and τ for weak field cases is preserved even for any fixed strong fields $\xi(t), Z(t), X(t)$ and $Y(t)$ for non-negative β to the extent that the term linear in $\omega(t)$ dominates the cosine and sine terms. The correspondence between τ and ω in (3.17) is quite complicated for finite $\omega(t)$ due to the presence of the derivatives of $\omega(t)$.

Thus (3.17) satisfies our criterion of BRST invariant path integral for any non-negative β. The relation (3.9) is recovered if one sets $\beta = 0$ in (3.17); the non-zero parameter $\beta \neq 0$ however renders a canonical structure of the theory better-defined. For example, the kinetic term for ghost fields in (3.13) disappears for $\beta = 0$. In this respect the gauge (3.5) is also analogous to the unitary gauge. In the following we set $\beta = \alpha > 0$ in (3.13),

$$\mathcal{L}_g = -\alpha\dot{B}\xi^{\omega} + B(Z^{\omega} - \lambda X^{\omega}) + \alpha i\dot{\bar{c}}\dot{c} - i\bar{c}(1 + g\lambda Y^{\omega})c + \frac{\alpha}{2}B^2 \quad (3.18)$$

and let $\alpha \to 0$ later. In the limit $\alpha = 0$, one recovers the gauge condition (3.5) defined in Ref.[8]. This procedure is analogous to R_ξ-gauge (or the ξ-limiting process of Lee and Yang[14])[15], where the (singular) unitary gauge is defined in the vanishing limit of the gauge parameter, $\xi \to 0$: In (3.18) the parameter α plays the role of ξ in R_ξ-gauge. In R_ξ-gauge one can keep $\alpha \neq 0$ without spoiling gauge invariance, and the advantage of this approach with $\alpha \neq 0$ is that one can avoid the appearance of (operator ordering) correction terms [8] when one moves from the Hamiltonian formalism to the

Lagrangian formalism and vice versa. This point will be discussed in detail when we analyze perturbative corrections to the ground state energy in Section 4.

By using the BRST invariance, one can show the λ- independence of (3.11) as follows:

$$\langle +\infty | -\infty \rangle_{\lambda+\delta\lambda} = \langle +\infty | -\infty \rangle_{\lambda} - \delta\lambda \frac{1}{\tilde{N}} \int d\mu [B(t) X^{\omega}(t) + ig\bar{c}(t) Y^{\omega}(t) c(t)]\; exp\{i \int (\mathcal{L}+\mathcal{L}_g) dt\} \tag{3.19}$$

where we perturbatively expanded in the variation of $\mathcal{L}_g$ for a change of the parameter $\lambda + \delta\lambda$,

$$\mathcal{L}_g(\lambda + \delta\lambda) = \mathcal{L}_g(\lambda) - \delta\lambda [B(t) X^{\omega}(t) + ig\bar{c}(t) Y^{\omega}(t) c(t)] \tag{3.20}$$

This expansion is justified since the normalization factor defined by (see eq.(2.10))

$$N \;\; = \;\; \int \mathcal{D}\tau \; exp\{-\frac{i}{2\alpha} \int \tau(x)^2 dt\} \tag{3.21}$$

is independent of λ provided that the global single-valuedness in (3.17) is satisfied. [See also the discussion related to eq.(5.1).] As was noted before, this path integral for N, which depends on α, is the only place where we explicitly encounter the Gribov-type copies in the present approach. By denoting the BRST transformed variables by prime, for example,

$$X^{\omega}(t)' = X^{\omega}(t) - ig\theta c(t) Y^{\omega}(t) \tag{3.22}$$

we have a BRST identity (or Slavnov-Taylor identity [16])

$$\begin{aligned}
&\frac{1}{\tilde{N}} \int d\mu \bar{c}(t) X^{\omega}(t) \; exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} \\
&= \frac{1}{\tilde{N}} \int d\mu' \bar{c}(t)' X^{\omega}(t)' \; exp\{i \int (\mathcal{L}' + \mathcal{L}_g') dt\} \\
&= \frac{1}{\tilde{N}} \int d\mu \bar{c}(t) X^{\omega}(t) \; exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\} \\
&+ \theta \frac{1}{\tilde{N}} \int d\mu [B(t) X^{\omega}(t) + ig\bar{c}(t) Y^{\omega}(t) c(t)] \; exp\{i \int (\mathcal{L} + \mathcal{L}_g) dt\}
\end{aligned} \tag{3.23}$$

where the first equality holds since the path integral is independent of the naming of integration variables provided that the asymptotic behavior and the boundary conditions are not modified by the change of variables. The second equality in (3.23) holds due to the BRST invariance of the measure and the action

$$\begin{aligned}
d\mu' &= d\mu, \\
\mathcal{L}' + \mathcal{L}_g' &= \mathcal{L} + \mathcal{L}_g
\end{aligned} \tag{3.24}$$

but

$$\bar{c}(t)'X^{\omega}(t)' = \bar{c}(t)X^{\omega}(t) + \theta[B(t)X^{\omega}(t) + ig\bar{c}(t)Y^{\omega}(t)c(t)] \tag{3.25}$$

From (3.23) one concludes

$$\frac{1}{\tilde{N}}\int d\mu[B(t)X^{\omega}(t) + ig\bar{c}(t)Y^{\omega}(t)c(t)]\; exp\{i\int(\mathcal{L} + \mathcal{L}_g)dt\} = 0 \tag{3.26}$$

and thus

$$\langle +\infty | -\infty \rangle_{\lambda+\delta\lambda} = \langle +\infty | -\infty \rangle_{\lambda} \tag{3.27}$$

in (3.19). This relation shows that the ground state energy is independent of the parameter λ; in particular one can choose $\lambda = 0$ in evaluating the ground state energy, which leads to the gauge condition (3.4) without Gribov complications.

In the path integral (3.11), one may impose *periodic* boundary conditions in time t on all the integration variables and let the time interval $\to \infty$ later so that BRST transformation (3.15) be consistent with the boundary conditions.

The analysis in this sub-section is general but formal. In the next sub-section we convert the path integral (3.11) to an operator Hamiltonian formalism and analyze in detail the structure of the ground state and the gauge independence of physical energy spectrum.

3.3, BRST ANALYSIS: OPERATOR HAMILTONIAN FORMALISM

We start with the BRST invariant effective Lagrangian

$$\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L} + \mathcal{L}_g \\
&= \frac{1}{2}\{[\dot{X}^{\omega}(t) + g\xi(t)^{\omega}Y^{\omega}(t)]^2 + [\dot{Y}^{\omega}(t) - g\xi^{\omega}(t)X^{\omega}(t)]^2 + [\dot{Z}^{\omega}(t) - \xi^{\omega}(t)]^2\} \\
&\quad -U[(X^{\omega}(t))^2 + (Y^{\omega}(t))^2] \\
&\quad -\alpha\dot{B}(t)\xi^{\omega}(t) + B(t)(Z^{\omega}(t) - \lambda X^{\omega}(t)) + \alpha i\dot{\bar{c}}(t)\dot{c}(t) \\
&\quad -i\bar{c}(t)[1 + g\lambda Y^{\omega}(t)]c(t) + \frac{\alpha}{2}B(t)^2
\end{aligned} \tag{3.28}$$

obtained from (3.1) and (3.18). A justification of (3.28), in particular its treatment of Gribov-type copies, rests on the path integral representation (3.11). In the following we suppress the suffix ω, which emphasizes that the BRST transformation is local in ω-space.

One can construct a Hamiltonian from (3.28) as

$$H = \frac{1}{2}[P_X^2 + P_Y^2 + P_Z^2] + U(X^2 + Y^2)$$

$$+\xi G - B(Z - \lambda X) + i\frac{1}{\alpha}p_c p_{\bar{c}} + i\bar{c}(1 + \lambda g Y)c - \frac{1}{2}\alpha B^2 \tag{3.29}$$

where

$$\begin{aligned}
P_X &= \frac{\partial}{\partial \dot{X}}\mathcal{L}_{eff} = \dot{X} + g\xi Y \\
P_Y &= \frac{\partial}{\partial \dot{Y}}\mathcal{L}_{eff} = \dot{Y} - g\xi X \\
P_Z &= \frac{\partial}{\partial \dot{Z}}\mathcal{L}_{eff} = \dot{Z} - \xi \\
P_B &= \frac{\partial}{\partial \dot{B}}\mathcal{L}_{eff} = -\alpha\xi \\
P_{\bar{c}} &= \frac{\partial}{\partial \dot{\bar{c}}}\mathcal{L}_{eff} = i\alpha\dot{c} \\
P_c &= \frac{\partial}{\partial \dot{c}}\mathcal{L}_{eff} = -i\alpha\dot{\bar{c}}
\end{aligned} \tag{3.30}$$

and the Gauss operator is given by

$$G \equiv g(XP_Y - YP_X) + P_Z \tag{3.31}$$

We note that $(p_{\bar{c}})^\dagger = -p_{\bar{c}}$ and $(p_c)^\dagger = -p_c$ from (3.30). The quantization condition

$$[P_B, B] = \frac{1}{i} \tag{3.32}$$

implies

$$[\xi, B] = \frac{i}{\alpha} \tag{3.33}$$

and thus one may take the representation

$$B = \frac{1}{i\alpha}\frac{\partial}{\partial \xi} \tag{3.34}$$

which is used later.

We also note the quantization conditions

$$\begin{aligned}
\{p_{\bar{c}}, \bar{c}\}_+ = \frac{1}{i} \quad &\rightarrow \quad \{\dot{c}, \bar{c}\}_+ = -\frac{1}{\alpha}, \\
\{p_c, c\}_+ = \frac{1}{i} \quad &\rightarrow \quad \{c, \dot{\bar{c}}\}_+ = \frac{1}{\alpha}
\end{aligned} \tag{3.35}$$

The BRST charge is obtained from $\mathcal{L}_{eff}$ (3.28) via the Noether current as

$$Q = cG - ip_{\bar{c}}B \tag{3.36}$$

The BRST charge Q is hermitian $Q^\dagger = Q$ and nil-potent

$$\{Q, Q\}_+ = 0 \tag{3.37}$$

by noting $\{c, c\}_+ = \{p_{\bar{c}}, p_{\bar{c}}\}_+ = \{p_{\bar{c}}, c\}_+ = 0$. The BRST transformation (3.15) is generated by Q, for example,

$$\begin{aligned} e^{-\theta Q} X(t) e^{\theta Q} &= X(t) - [\theta Q, X(t)] \\ &= X(t) - i\theta g c(t) Y(t), \\ e^{-\theta Q} \bar{c}(t) e^{\theta Q} &= \bar{c}(t) - [\theta Q, \bar{c}(t)] \\ &= \bar{c}(t) + \theta B(t) \end{aligned} \tag{3.38}$$

by noting $\theta^2 = 0$.

Some of the BRST invariant physical operators are given by

$$\begin{aligned} &X^2 + Y^2, \quad P_X^2 + P_Y^2, \\ &L_Z = X P_Y - Y P_X \end{aligned} \tag{3.39}$$

The Hamiltonian in (3.29) is rewritten by using the BRST charge as

$$H = H_0 + i\{Q, \xi p_c\}_+ + \{Q, \bar{c}(Z - \lambda X\}_+ + \frac{\alpha}{2}\{Q, B\bar{c}\}_+ \tag{3.40}$$

with

$$H_0 \equiv \frac{1}{2}[P_X^2 + P_Y^2 + P_Z^2] + U(X^2 + Y^2) \tag{3.41}$$

One defines a physical state Ψ as an element of BRST cohomology

$$\Psi \in \ Ker\ Q/Im\ Q \tag{3.42}$$

namely

$$Q\Psi = 0 \tag{3.43}$$

but Ψ is *not* written in a form $\Psi = Q\Phi$ with a non-vanishing Φ.

The time development of Ψ is dictated by Schroedinger equation

$$i\frac{\partial}{\partial t}\Psi(t) = H\Psi(t) \tag{3.44}$$

and thus

$$\begin{aligned}\Psi(\delta t) &= e^{-iH\delta t}\Psi(0) \\ &= \Psi(0) - i\delta t H\Psi(0) \\ &= \Psi(0) - i\delta t H_0\Psi(0) \\ &\quad -i\delta t Q\{i\xi p_c + \bar{c}(Z-\lambda X) + \frac{\alpha}{2}B\bar{c}\}\Psi(0) \\ &\simeq \Psi(0) - i\delta t H_0\Psi(0)\end{aligned} \tag{3.45}$$

in the sense of BRST cohomology by noting (3.40) and $Q\Psi(0) = 0$. Note that the Hamiltonian is BRST invariant

$$[Q, H_0] = [Q, H] = 0 \tag{3.46}$$

If one solves the time independent Schroedinger equation

$$H_0\Psi(0) = E\Psi(0) \tag{3.47}$$

with $Q\Psi(0) = 0$, one obtains

$$\begin{aligned}\Psi(0)^\dagger e^{-iHt}\Psi(0) &= e^{-iEt}\Psi(0)^\dagger\Psi(0) \\ &= e^{-iEt}\end{aligned} \tag{3.48}$$

by noting $\Psi(0)^\dagger Q = 0$. The eigen-value equation (3.47) is gauge independent and thus E is formally gauge independent, but the value of E is constrained by $Q\Psi(0) = 0$ and thus we need a more detailed analysis.

The basic task in the present BRST approach is to construct physical states Ψ satisfying (3.42). We construct such physical states, in particular the ground state, as Fock states. For an explicit construction of physical states, we use the harmonic potential considered in Ref.[8]

$$U(X^2+Y^2) = \frac{\omega^2}{2}(X^2+Y^2) \tag{3.49}$$

and we write H in (3.29) as

$$\begin{aligned}H &= \frac{1}{2}[P_X^2 + P_Y^2] + \frac{\omega^2}{2}(X^2+Y^2) + \frac{1}{2}(gL_Z)^2 \\ &\quad +\frac{1}{2}(\tilde{P}_Z + \tilde{\xi})^2 + \frac{1}{2\alpha}Z^2 - \frac{1}{2}\tilde{\xi}^2 - \frac{1}{2\alpha}(\alpha B + Z)^2 \\ &\quad +\frac{i}{\alpha}p_c p_{\bar{c}} + i\bar{c}c - \lambda\{Q, \bar{c}X\}_+\end{aligned} \tag{3.50}$$

with

$$\begin{aligned}
\tilde{P}_Z &\equiv P_Z + gL_Z = G \\
\tilde{\xi} &\equiv \xi - gL_Z
\end{aligned} \tag{3.51}$$

Eq.(3.50) indicates that the freedom associated with Z has positive norm but the freedom $\tilde{\xi}$ has negative norm. We thus define

$$\begin{aligned}
X &= \frac{1}{\sqrt{2\omega}}(a_X + a_X^\dagger), \\
P_X &= i\sqrt{\frac{\omega}{2}}(-a_X + a_X^\dagger), \\
Y &= \frac{1}{\sqrt{2\omega}}(a_Y + a_Y^\dagger), \\
P_Y &= i\sqrt{\frac{\omega}{2}}(-a_Y + a_Y^\dagger), \\
Z &= \frac{1}{\sqrt{2\nu}}(d + d^\dagger), \\
\tilde{P}_Z &= P_Z + gL_Z = i\sqrt{\frac{\nu}{2}}[(b-d) - (b-d)^\dagger], \\
\tilde{\xi} &= \xi - gL_Z = (-i)\sqrt{\frac{\nu}{2}}(b - b^\dagger), \\
\alpha B + Z &= \frac{1}{\sqrt{2\nu}}(b + b^\dagger), \\
c &= \frac{1}{\sqrt{2\mu}}(\hat{c} + \hat{c}^\dagger), \\
p_{\bar{c}} &= \frac{\mu}{\sqrt{2}}(\hat{c} - \hat{c}^\dagger), \\
\bar{c} &= \frac{1}{\sqrt{2\mu}}(\hat{\bar{c}} + \hat{\bar{c}}^\dagger), \\
p_c &= \frac{\mu}{\sqrt{2}}(-\hat{\bar{c}} + \hat{\bar{c}}^\dagger)
\end{aligned} \tag{3.52}$$

with

$$\nu \equiv \sqrt{\frac{1}{\alpha}}, \quad \mu = \sqrt{\frac{1}{\nu}} \tag{3.53}$$

In (3.52) the operator expansion of X, Y, c, and $\bar{c}$ is the standard one, but the expansion of $\tilde{P}_Z, \tilde{\xi}$ and $\alpha B + Z$ is somewhat unconventional.

The canonical commutators are satisfied by postulating

$$[a_X, a_X^\dagger] = 1,$$

$$[a_Y, a_Y^\dagger] = 1,$$

$$[d, d^\dagger] = 1,$$

$$\begin{aligned}
[\bar{b}, \bar{b}^\dagger] &= -1, \\
\{\hat{c}, \hat{\bar{c}}^\dagger\}_+ &= -i, \\
\{\hat{c}^\dagger, \hat{\bar{c}}\}_+ &= i
\end{aligned} \tag{3.54}$$

and all other commutators are vanishing. Here we defined

$$b \equiv \bar{b} - i\frac{1}{\sqrt{2\nu}} gL_Z \tag{3.55}$$

so that

$$\begin{aligned}
P_Z &= i\sqrt{\frac{\nu}{2}}[(\bar{b} - d) - (\bar{b} - d)^\dagger], \\
\xi &= (-i)\sqrt{\frac{\nu}{2}}(\bar{b} - \bar{b}^\dagger), \\
\alpha B + Z &= \frac{1}{\sqrt{2\nu}}(\bar{b} + \bar{b}^\dagger)
\end{aligned}$$

The operator b also satisfies

$$[b, b^\dagger] = -1 \tag{3.56}$$

The variables $\bar{b}$ in (3.54) and b in (3.56) carry negative norm. The Hamiltonian (3.50) is then written as

$$\begin{aligned}
H &= \omega(a_X^\dagger a_X + a_Y^\dagger a_Y + 1) + \frac{1}{2}(gL_Z)^2 \\
&\quad +\nu[d^\dagger d - b^\dagger b + 1] + i\nu[\hat{\bar{c}}^\dagger \hat{c} - \hat{c}^\dagger \hat{\bar{c}} + i] \\
&\quad +\lambda\{Q, \bar{c}X\}_+
\end{aligned} \tag{3.57}$$

with the BRST charge

$$\begin{aligned}
Q &= cG - ip_{\bar{c}}B \\
&= i\nu[\hat{c}^\dagger(b - d) - \hat{c}(b - d)^\dagger] \\
&= i\nu[\hat{c}^\dagger(\bar{b} - d) - \hat{c}(\bar{b} - d)^\dagger] + \sqrt{\frac{\nu}{2}}(\hat{c}^\dagger + \hat{c})gL_Z
\end{aligned} \tag{3.58}$$

and

$$L_Z = XP_Y - YP_X = i(a_X^\dagger a_Y - a_Y^\dagger a_X) \tag{3.59}$$

The BRST charge in (3.58) is nil-potent

$$Q^2 = \nu^2 \hat{c}^\dagger \hat{c}[b - d, b^\dagger - d^\dagger] = 0 \tag{3.60}$$

by noting (3.54) and (3.56).

We thus define the (physical) ground state at $t=0$ by

$$
\begin{aligned}
b|0\rangle &= d|0\rangle = 0, \\
\hat{c}|0\rangle &= \hat{\bar{c}}|0\rangle = 0, \\
a_X|0\rangle &= a_Y|0\rangle = 0
\end{aligned}
\tag{3.61}
$$

which ensures

$$
Q|0\rangle = 0 \tag{3.62}
$$

The zero-point energy of Z and ξ and the zero-point energy of c and $\bar{c}$ in (3.57) cancel each other for the state $|0\rangle$. When one defines a unitary transformation [8]

$$
\begin{aligned}
a_X &= \frac{1}{\sqrt{2}}(\tilde{a}_X - i\tilde{a}_Y) \\
a_Y &= \frac{1}{\sqrt{2}}(-i\tilde{a}_X + \tilde{a}_Y)
\end{aligned}
\tag{3.63}
$$

we can write the physical part of H in (3.57) as

$$
\begin{aligned}
H_{phys} &\equiv \omega(a_X^\dagger a_X + a_Y^\dagger a_Y + 1) + \frac{1}{2}(gL_Z)^2 \\
&= \omega(\tilde{a}_X^\dagger \tilde{a}_X + \tilde{a}_Y^\dagger \tilde{a}_Y + 1) + \frac{g^2}{2}(\tilde{a}_X^\dagger \tilde{a}_X - \tilde{a}_Y^\dagger \tilde{a}_Y)^2
\end{aligned}
\tag{3.64}
$$

If one recalls the relation

$$
\begin{aligned}
b &= \frac{1}{\sqrt{2\nu}}[\nu\frac{\partial}{i\partial\xi} + \nu Z + i\xi] \\
b - d &= \frac{1}{\sqrt{2\nu}}[-i(P_Z + gL_Z) + \nu\frac{\partial}{i\partial\xi}]
\end{aligned}
\tag{3.65}
$$

where we used $\alpha B = \frac{\partial}{i\partial\xi}$ in (3.34), the state $|0\rangle$ in (3.61) is required to satisfy

$$
\begin{aligned}
\{\nu\frac{\partial}{\partial\xi} - \xi + i\nu Z\}|0\rangle_{Z\xi} = 0, \\
\{\nu\frac{\partial}{\partial\xi} + \frac{1}{i}\frac{\partial}{\partial Z} + gL_Z\}|0\rangle_{Z\xi} = 0
\end{aligned}
\tag{3.66}
$$

Eq.(3.66) has a solution

$$
\begin{aligned}
|0, L_Z\rangle_{Z\xi} &= N_0 \exp\{\frac{1}{2\nu}\xi^2 - i\xi Z - igL_Z Z - \frac{\nu}{2}Z^2\} \\
&= N_0 \exp\{\frac{1}{2\nu}(\xi - i\nu Z)^2 - igL_Z Z\}
\end{aligned}
\tag{3.67}
$$

which depends on L_Z ; N_0 is a normalization constant. The inner product of this state needs to be defined by means of a 90 degree rotation in the variable ξ

$$\langle 0, L_Z|0, L_Z\rangle = N_0^2 \int dZ d\xi e^{\frac{1}{\nu}\xi^2 - \nu Z^2} \tag{3.68}$$

which reflects the fact that the ξ-variable carries negative norm. If one projects the state $|0, L_Z\rangle_{Z\xi}$ to the one with $p_\xi = 0$ by Fourier transformation, which is the general procedure of Dirac in his treatment of singular Lagrangian [17], one obtains

$$\int d\xi |0, L_Z\rangle_{Z\xi} \sim e^{-igL_Z Z} \tag{3.69}$$

This is the physical ground state naively expected for (Z, P_Z) system for a given L_Z on the basis of invariance under the Gauss operator in (3.31), and it is independent of the gauge parameter $\nu = \sqrt{\frac{1}{\alpha}}$.

Other sectors of the ground state $|0\rangle_{XY\bar{c}c}$ are constructed in a standard manner

$$\tilde{a}_X|0\rangle_{XY\bar{c}c} = \tilde{a}_Y|0\rangle_{XY\bar{c}c} = \hat{c}|0\rangle_{XY\bar{c}c} = \hat{\bar{c}}|0\rangle_{XY\bar{c}c} = 0 \tag{3.70}$$

and the entire ground state in (3.62) is written as

$$|0\rangle = |0, L_Z\rangle_{Z\xi} \otimes |0\rangle_{XY\bar{c}c} \tag{3.71}$$

where $L_Z = \tilde{a}_X^\dagger \tilde{a}_X - \tilde{a}_Y^\dagger \tilde{a}_Y$ is replaced by a c-number by acting L_Z on $|0\rangle_{XY\bar{c}c}$. In the present case, $L_Z = 0$.

The ground state thus defined has a time dependence described by H in (3.57)

$$\begin{aligned} e^{-iH\delta t}|0\rangle &= |0\rangle - i\omega\delta t|0\rangle + \delta t\lambda Q\bar{c}X|0\rangle \\ &\simeq |0\rangle - i\omega\delta t|0\rangle \end{aligned} \tag{3.72}$$

in the sense of BRST cohomology by noting (3.62). Thus we can *represent* the vacuum (or ground) state in $Ker\ Q/Im\ Q$ by $|0\rangle$ in (3.71) for any time, and we have

$$\langle 0|e^{-iHt}|0\rangle = e^{-i\omega t} \tag{3.73}$$

Excited physical states are represented by

$$\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle \equiv |0, L_Z = n_1 - n_2\rangle_{Z\xi} \otimes \frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle_{XY\bar{c}c} \tag{3.74}$$

One obtains the eigenvalue of H in (3.57) and (3.64) as

$$\begin{aligned}H\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle &= [\omega(n_1+n_2+1)+\frac{g^2}{2}(n_1-n_2)^2]\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle \\ &\quad +\lambda Q\bar{c}X\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle \\ &\simeq [\omega(n_1+n_2+1)+\frac{g^2}{2}(n_1-n_2)^2]\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle\end{aligned} \tag{3.75}$$

in the sense of BRST cohomology, since

$$Q\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle = 0 \tag{3.76}$$

This (3.76) is confirmed by using the expression of Q in (3.58) as

$$\begin{aligned}&e^{-\theta Q}\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle \\ &= e^{-\theta i\nu\hat{c}^\dagger b}\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle \\ &= e^{-\theta i\nu\hat{c}^\dagger\bar{b}}|0, L_Z=n_1-n_2\rangle_{Z\xi}\otimes e^{-\theta\sqrt{\frac{\nu}{2}}\hat{c}^\dagger gL_Z}\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle_{XYc\bar{c}} \\ &= e^{\theta\sqrt{\frac{\nu}{2}}\hat{c}^\dagger g(n_1-n_2)}|0, L_Z=n_1-n_2\rangle_{Z\xi}\otimes e^{-\theta\sqrt{\frac{\nu}{2}}\hat{c}^\dagger g(n_1-n_2)}\frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle_{XYc\bar{c}} \\ &= \frac{1}{\sqrt{n_1!n_2!}}(\tilde{a}_X^\dagger)^{n_1}(\tilde{a}_Y^\dagger)^{n_2}|0\rangle\end{aligned} \tag{3.77}$$

where we used

$$\begin{aligned}L_Z &= \tilde{a}_X^\dagger\tilde{a}_X-\tilde{a}_Y^\dagger\tilde{a}_Y, \\ b|0, L_Z=n_1-n_2\rangle_{Z\xi} &= \{\bar{b}-i\frac{1}{\sqrt{2\nu}}g(n_1-n_2)\}|0, L_Z=n_1-n_2\rangle_{Z\xi} \\ &= 0\end{aligned} \tag{3.78}$$

The second relation in (3.78) is regarded as a constraint on the ground state of $\bar{b}$ for a given value of L_Z,which is a manifestation of the Gauss constraint in the present formulation.

The states which include $b^\dagger, d^\dagger, \hat{c}^\dagger$,and $\hat{\bar{c}}^\dagger$ excitations become unphysical and are removed by BRST cohomology. For example, the state

$$-\nu(b-d)^\dagger|0\rangle = Q\hat{\bar{c}}^\dagger|0\rangle \tag{3.79}$$

is BRST invariant and has energy (for $\lambda=0$)

$$HQ\hat{\bar{c}}^\dagger|0\rangle = (\omega+\nu)Q\hat{\bar{c}}^\dagger|0\rangle \tag{3.80}$$

but it is obviously excluded by BRST cohomology. The unphysical excitations $b^\dagger, d^\dagger, \hat{c}^\dagger$,and $\hat{\bar{c}}^\dagger$ form the components of *non-trivial* BRST superfields $Z^\omega(t,\theta)$ and $\bar{c}(t,\theta)$ in (3.15),

$$
\begin{aligned}
Z^\omega(t,\theta) &= Z^\omega(t) + i\theta c(t) \\
&= \frac{1}{\sqrt{2\nu}}(d + d^\dagger) + i\theta \frac{1}{\sqrt{2\mu}}(\hat{c} + \hat{c}^\dagger) \\
\bar{c}(t,\theta) &= \bar{c}(t) + \theta B(t) \\
&= \frac{1}{\sqrt{2\mu}}(\hat{\bar{c}} + \hat{\bar{c}}^\dagger) + \theta\nu\sqrt{\frac{\nu}{2}}[b - d + (b-d)^\dagger]
\end{aligned}
\tag{3.81}
$$

A characteristic property of these non-trivial superfields is that the second components of the superfields, which are BRST transform of the first components, contain terms *linear* in the elementary field. The basic theorem of BRST symmetry is that any BRST invariant state which contains those unphysical degrees of freedom ,$b^\dagger, d^\dagger, \hat{c}^\dagger$ and $\hat{\bar{c}}^\dagger$, is written in the BRST exact form such as in (3.79) and thus it is removed by BRST cohomology. See Ref.[13].

The present BRST analysis is in accord with an explicit construction of physical states in Ref.[8]. One can safely take the limit $\alpha \to 0$ (or $\nu = \frac{1}{\sqrt{\alpha}} \to \infty$) in the physical sector, though unphysical excitations such as in (3.80) acquire infinite excitation energy in this limit just like unphysical excitations in gauge theory defined by R_ξ-gauge[15].

4 Perturbative calculation in path integral

It has been shown in [8] that the correction terms arising from operator ordering plays a crucial role in the evaluation of perturbative corrections to ground state energy in Lagrangian path integral formula. This problem is often treated casually in conventional perturbative calculations; a general belief (and hope) is that Lorentz invariance and BRST invariance somehow takes care of the operator ordering problem. In the following, we show that BRST invariance and $T^\star$-product prescription reproduce the correct result of Ref.[8] provided that one uses a canonically well-defined gauge such as R_ξ-gauge with $\alpha \neq 0$ in (3.18). This check is important to establish the equivalence of (3.11) to the path integral formula in Ref.[8]. See also Ref.[18]. If one starts with $\alpha = 0$ from the on-set, one needs correction terms calculated in Ref.[8].

To be precise, what we want to evaluate is eq.(3.11), namely

$$\langle +\infty | -\infty \rangle = \frac{1}{\tilde{N}} \int d\mu \; exp\{iS(X^\omega, Y\omega, Z^\omega, \xi^\omega) + i \int \mathcal{L}_g dt\} \tag{4.1}$$

We define the path integral for a sufficiently large time interval

$$t \in [T/2, -T/2] \tag{4.2}$$

and let $T \to \infty$ later. In the actual calculation, there appear two important aspects which need to be taken into account:

(i) We impose periodic boundary conditions on all the variables so that BRST transformation (3.15) is well-defined including the boundary conditions.

(ii) In the actual evaluation of the path integral as well as Feynman diagrams , we may apply the Wick rotation and perform Euclidean calculations.

The exact ground state energy of (4.1) is given by eq.(3.73) as

$$E = \omega \tag{4.3}$$

Namely, we have no correction depending on the gauge parameter λ and the coupling constant g. As was already shown in (3.27), the absence of λ dependence is a result of BRST symmetry. This property is thus more general and, in fact, it holds for all the energy spectrum of physical states; this can be shown by using the Schwinger's action principle [18] and the definition of physical states in (3.42). The perturbative check of λ-independence or Slavnov-Taylor identities in general is carried out in the standard manner. On the other hand, the absence of g- dependence is an effect of more dynamical origin. In this Section we concentrate on the evaluation of g-dependence by taking a view that the λ- independence has been generally established in (3.27).

We first perform the Gaussian path integral over B-variable by noting

$$\begin{aligned} \mathcal{L}_{gauge} &= \frac{\alpha}{2}B^2 + \alpha B\dot{\xi} + B(Z - \lambda X) \\ &= \frac{\alpha}{2}[B + \frac{1}{\alpha}(\alpha\dot{\xi} + Z - \lambda X)]^2 - \frac{1}{2\alpha}[\alpha\dot{\xi} + Z - \lambda X]^2 \\ &\Rightarrow -\frac{1}{2\alpha}[\alpha\dot{\xi} + Z - \lambda X]^2 \end{aligned} \tag{4.4}$$

We thus consider the path integral

$$\langle +\infty | -\infty \rangle = \frac{1}{N}\int \mathcal{D}X\mathcal{D}Y\mathcal{D}Z\mathcal{D}\xi\mathcal{D}\bar{c}\mathcal{D}c \; exp\{i\int \mathcal{L}_{eff}dt\} \tag{4.5}$$

where N is the original normalization constant in (3.21), and

$$\begin{aligned}
\mathcal{L}_{eff} &= \frac{1}{2}\{[\dot{X}(t) + g\xi(t)Y(t)]^2 + [\dot{Y}(t) - g\xi(t)X(t)]^2\} - \frac{\omega^2}{2}[X(t)^2 + Y(t)^2] \\
&\quad + \frac{1}{2}\dot{Z}(t)^2 - \frac{1}{2\alpha}[Z(t) - \lambda X(t)]^2 - \frac{\alpha}{2}\dot{\xi}(t)^2 + \frac{1}{2}\xi(t)^2 \\
&\quad + \lambda\dot{\xi}(t)X(t) + \alpha i\dot{\bar{c}}(t)\dot{c}(t) - i\bar{c}(t)(1 + g\lambda Y(t))c(t) \\
&\equiv \mathcal{L}_0 + \mathcal{L}_I
\end{aligned} \tag{4.6}$$

with

$$\begin{aligned}
\mathcal{L}_0 &\equiv \frac{1}{2}[\dot{X}(t)^2 + \dot{Y}(t)^2] - \frac{\omega^2}{2}[X(t)^2 + Y(t)^2] \\
&\quad + \frac{1}{2}\dot{Z}(t)^2 - \frac{1}{2\alpha}Z(t)^2 - \frac{\alpha}{2}\dot{\xi}(t)^2 + \frac{1}{2}\xi(t)^2 \\
&\quad + \alpha i\dot{\bar{c}}(t)\dot{c}(t) - i\bar{c}(t)c(t),
\end{aligned} \tag{4.7}$$

$$\mathcal{L}_I \equiv g\xi(t)[\dot{X}(t)Y(t) - \dot{Y}(t)X(t)] + \frac{1}{2}g^2\xi(t)^2[X(t)^2 + Y(t)^2] \tag{4.8}$$

where we set $\lambda = 0$ in the final expressions in (4.7) and (4.8).

The propagators are defined by $\mathcal{L}_0$ in (4.7) in the standard manner as

$$\begin{aligned}
\langle T^\star X(t_1)X(t_2)\rangle &= \langle T^\star Y(t_1)Y(t_2)\rangle \\
&= \int \frac{dk}{2\pi}e^{ik(t_1-t_2)}\frac{i}{k^2 - \omega^2 + i\epsilon} \\
\langle T^\star Z(t_1)Z(t_2)\rangle &= \int \frac{dk}{2\pi}e^{ik(t_1-t_2)}\frac{i}{k^2 - 1/\alpha + i\epsilon} \\
\langle T^\star \xi(t_1)\xi(t_2)\rangle &= \int \frac{dk}{2\pi}e^{ik(t_1-t_2)}\frac{-i/\alpha}{k^2 - 1/\alpha + i\epsilon} \\
\langle T^\star \bar{c}(t_1)c(t_2)\rangle &= \int \frac{dk}{2\pi}e^{ik(t_1-t_2)}\frac{-1/\alpha}{k^2 - 1/\alpha + i\epsilon}
\end{aligned} \tag{4.9}$$

where we took $T \to \infty$ limit in the evaluation of those propagators; this procedure is justified for the evaluation of corrections to the ground state energy since all the momentum integrations in (4.11) below are well-convergent.

Up to the second order of perturbation in $\mathcal{L}_I$,we have

$$\langle +\infty | -\infty \rangle = \frac{1}{N}\int d\mu \; e^{i\int \mathcal{L}_0 dt}\{1 + i\int dt\langle T^\star \frac{1}{2}g^2\xi(t)^2[X(t)^2 + Y(t)^2]\rangle$$

$$
\begin{aligned}
&+ \quad \frac{(i)^2}{2!}\int dt_1 dt_2 \langle T^\star g^2 \xi(t_1)\xi(t_2)[\dot{X}(t_1)Y(t_1)-\dot{Y}(t_1)X(t_1)] \\
&\qquad\qquad\qquad\qquad \times[\dot{X}(t_2)Y(t_2)-\dot{Y}(t_2)X(t_2)]\rangle\} \\
&= \quad \frac{1}{N}\int d\mu\ e^{i\int \mathcal{L}_0 dt}\{1 + ig^2\int dt \langle T^\star \xi(t)\xi(t)\rangle\langle T^\star X(t)X(t)\rangle \\
&+ \quad (i)^2 g^2 \int dt_1 dt_2 \langle T^\star \xi(t_1)\xi(t_2)\rangle\langle T^\star \dot{X}(t_1)\dot{X}(t_2)\rangle\langle T^\star Y(t_1)Y(t_2)\rangle \\
&- \quad (i)^2 g^2 \int dt_1 dt_2 \langle T^\star \xi(t_1)\xi(t_2)\rangle\langle T^\star \dot{X}(t_1)X(t_2)\rangle\langle T^\star Y(t_1)\dot{Y}(t_2)\rangle\}
\end{aligned} \tag{4.10}
$$

by taking the symmetry in X and Y into account.

The terms of order g^2 in (4.10) are written by using the propagators in (4.9) as

$$
\begin{aligned}
&(-\frac{i}{\alpha}g^2)\int dt \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{1}{l^2+\omega^2} \\
&+(\frac{i}{\alpha}g^2)\int dt \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{l^2}{l^2+\omega^2}\frac{1}{(k+l)^2+\omega^2} \\
&+(\frac{i}{\alpha}g^2)\int dt \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{1}{l^2+\omega^2}\frac{l(l+k)}{(k+l)^2+\omega^2} \\
= \ &(-\frac{i}{\alpha}g^2)T \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{1}{l^2+\omega^2} \\
&+(\frac{i}{\alpha}g^2)T \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{1}{l^2+\omega^2} \\
&+(-\frac{i}{\alpha}g^2)T \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{\omega^2}{l^2+\omega^2}\frac{1}{(k+l)^2+\omega^2} \\
&+(\frac{i}{\alpha}g^2)T \int \frac{dk}{2\pi}\frac{1}{k^2+1/\alpha}\int\frac{dl}{2\pi}\frac{1}{l^2+\omega^2}\frac{l(l+k)}{(k+l)^2+\omega^2}
\end{aligned} \tag{4.11}
$$

after the Wick rotation. We here note that the $T^\star$-product prescription is crucial in obtaining (4.11) ; the $T^\star$-product commutes with the time derivative operation, which is intuitively understood from the fact that the basic path integration variables are field variables (and not their time derivatives) in the Lagrangian path integral formula [19]. In this approach, the conventional T-product is defined from $T^\star$-product via the Bjorken-Johnson-Low prescription [20].

The first two terms in (4.11) cancel each other. The last two terms in (4.11) can be evaluated as follows: In the third term in (4.11), one can rewrite the integrand as

$$
\begin{aligned}
\frac{\omega^2}{l^2+\omega^2}\frac{1}{(k+l)^2+\omega^2} &= (\frac{1}{2i})^2(\frac{-1}{l+i\omega}+\frac{1}{l-i\omega})(\frac{-1}{l+k+i\omega}+\frac{1}{l+k-i\omega}) \\
&\rightarrow (\frac{1}{2i})^2\{\frac{-1}{l+i\omega}\frac{1}{l+k-i\omega}+\frac{1}{l-i\omega}\frac{-1}{l+k+i\omega}\} \\
&= (\frac{1}{2})^2\{\frac{1}{l+i\omega}\frac{1}{l+k-i\omega}+\frac{1}{l-i\omega}\frac{1}{l+k+i\omega}\}
\end{aligned} \tag{4.12}
$$

since the poles displaced in the same side of the real axis do not contribute to the integral over $\int dl$; the contour can be shrunk to zero without encircling poles. Similarly, in the last term in (4.11),

$$\begin{aligned}\frac{1}{l^2+\omega^2}\frac{l(l+k)}{(k+l)^2+\omega^2} &= (\frac{1}{2})^2(\frac{1}{l+i\omega}+\frac{1}{l-i\omega})(\frac{1}{l+k+i\omega}+\frac{1}{l+k-i\omega}) \\ &\to (\frac{1}{2})^2\{\frac{1}{l+i\omega}\frac{1}{l+k-i\omega}+\frac{1}{l-i\omega}\frac{1}{l+k+i\omega}\} \qquad (4.13)\end{aligned}$$

The last two terms in (4.11) thus cancel each other, and we obtain only the lowest order contribution in (4.11)

$$\begin{aligned}\langle +\infty | -\infty\rangle &= \frac{1}{N}\int d\mu\ e^{i\int \mathcal{L}_0 dt} \\ &= const \times \frac{1}{[2i\sin(\omega T/2)]^2}\frac{1}{N}\frac{det[\alpha\partial_t^2+1]}{\sqrt{det[\partial_t^2+1/\alpha]det[\alpha\partial_t^2+1]}} \\ &= const \times \frac{1}{[2i\sin(\omega T/2)]^2} \quad for\ T\to\infty, \qquad (4.14)\end{aligned}$$

where the determinant factors coming from $Z, \xi, \bar{c}$ and c integration combined with the normalization constant N cancel completely among themselves. The last expression in (4.14) is a standard path integral of harmonic oscillators X and Y with periodic boundary conditions [21], and it may be expanded as

$$const \times \frac{1}{[2i\sin(\omega T/2)]^2} = const \times e^{-i\omega T}(1+\sum_{n_1,n_2=0}^{\infty} e^{-i\omega T(n_1+n_2)}) \qquad (4.15)$$

for $T\to\infty$, where the summation over the non-negative integers n_1 and n_2 excludes the case $n_1=n_2=0$. To be precise, the T-dependence of the normalization constant N needs to be taken into account to obtain the last expression of (4.14) [21]. The ground state energy is then obtained from

$$\begin{aligned}\langle +\infty | -\infty\rangle &= \lim_{T\to\infty}\langle 0|e^{-iH[T/2-(-T/2)]}|0\rangle \\ &= \lim_{T\to\infty} const \times e^{-i\omega T} \qquad (4.16)\end{aligned}$$

which is justified for $T=-iT_E$ and $T_E\to\infty$ in Euclidean theory. We thus obtain the ground state energy

$$E=\omega \qquad (4.17)$$

to be consistent with (3.73).

5 Discussion and conclusion

The BRST symmetry plays a central role in modern gauge theory, and the BRST invariant path integral can be formulated by summing over all the Gribov-type copies in a very specific manner provided that the crucial correspondence in (2.11) or (3.17) is globally single valued[5]. This criterion is satisfied by the soluble gauge model proposed in Ref.[8], and it is encouraging that the BRST invariant prescription is in accord with the canonical analysis of the soluble gauge model in Ref.[8]. The detailed explicit analysis in Ref.[8] and the present somewhat formal BRST analysis are complementary to each other. In Ref.[8], the problem related to the so-called Gribov horizon , in particular the possible singularity associated with it, has been analyzed in greater detail; this is crucial for the analysis of more general situation. On the other hand, an advantage of the BRST analysis is that one can clearly see the gauge independence of physical quantities such as the energy spectrum as a result of BRST identity.

The BRST approach allows a transparent treatment of general class of gauge conditions implemented by (3.18). This gauge condition with $\alpha \neq 0$ renders the canonical structure better-defined, and it allows simpler perturbative treatments of the problems such as the corrections to the ground state energy. Our calculation vis-a-vis the explicit canonical analysis in Ref.[8] may provide a (partial) justification of conventional covariant perturbation theory in gauge theory, which is based on Lorentz invariance(or $T^{\star}$-product) and BRST invariance without the operator ordering terms.

Motivated by the observation in Ref.[8] to the effect that the Gribov horizons are not really singular in quantum mechanical sense, which is in accord with our path integral in (3.21), we would like to make a speculative comment on the role of Gribov copies in QCD. First of all, the topological phenomena related to instantons for which the Coulomb gauge is generally singular may be analyzed in the temporal gauge $A_0 = 0$; this gauge is relatively free from the Gribov complications[3]. A semi-classical treatment of instantons with small quantum fluctuations around them will presumably give qualitatively reliable estimates of topological effects.

As for other non-perturbative effects such as quark confinement and hadronic spectrum, one may follow the argument of Witten [22] on the basis of $1/N$ expansion in

QCD[23]; he argues that the $1/N$ expansion scheme comes closer to the real QCD than the instanton analysis in the study of hadron spectrum. If this is the case, one can analyze the qualitative aspects of hadron spectrum on the basis of a sum of (an infinite number of) Feynman diagrams. This diagramatic approach or an analytical treatment equivalent to it in the Feynman-type gauge deals with topologically trivial gauge fields but may still suffer from the Gribov copies, as is suggested by the analysis in Ref.[4]: If one assumes that the vacuum is unique in this case as is the case in L^2-space, the global single-valuedness in (2.11) in the context of path integral (2.10) will be preserved for infinitesimally small fields A_μ. By a continuity argument, a smooth deformation of A_μ in L^2-space (or its extension as explained in Section 2) will presumably keep the integral (2.10) unchanged. If this argument should be valid, our path integral formula in (2.4) would be justified. If our speculation is correct, the formal path integral formula (2.5) will provide a basis for the analysis of some non-perturbative aspects of QCD.

On the other hand, the Gribov problem may also suggest the presence of some field configurations which do not satisfy any given gauge condition in four dimensional non-Abelian gauge theory [2]. For example, one may not be able to find any gauge parameter $\omega(x)$ which satisfies

$$\partial^\mu A_\mu^\omega(x) = 0 \tag{5.1}$$

for some fields A_μ. Although the measure of such field configurations in path integral is not known, the presence of such filed configurations would certainly modify the asymptotic correspondence in (2.11). In the context of BRST symmetry, the Gribov problem may then induce complicated phenomena such as the dynamical instability of BRST symmetry[24]. If the dynamical instability of BRST symmetry should take place, the relation corresponding to (3.27), which is a result of the BRST invariance of the vacuum, would no longer be derived. In the framework of path integral, this failure of (3.27) would be recognized as the failure of the expansion (3.19) since the normalization factor N in (3.21) would generally depend on not only field variables but also λ if the global single-valuedness in (3.17) should be violated.

Finally, we note that the lattice gauge theory [25], which is based on compactified field variables, is expected to change the scope and character of the Gribov problem completely. The Gribov problem is intricately related to the difficult issue of the non-perturbative

continuum limit of lattice gauge theory.[Note that the perturbative continuum limit is not quite relevant in the present context, since we do not worry about the Gribov problem much in weak field perturbation theory].

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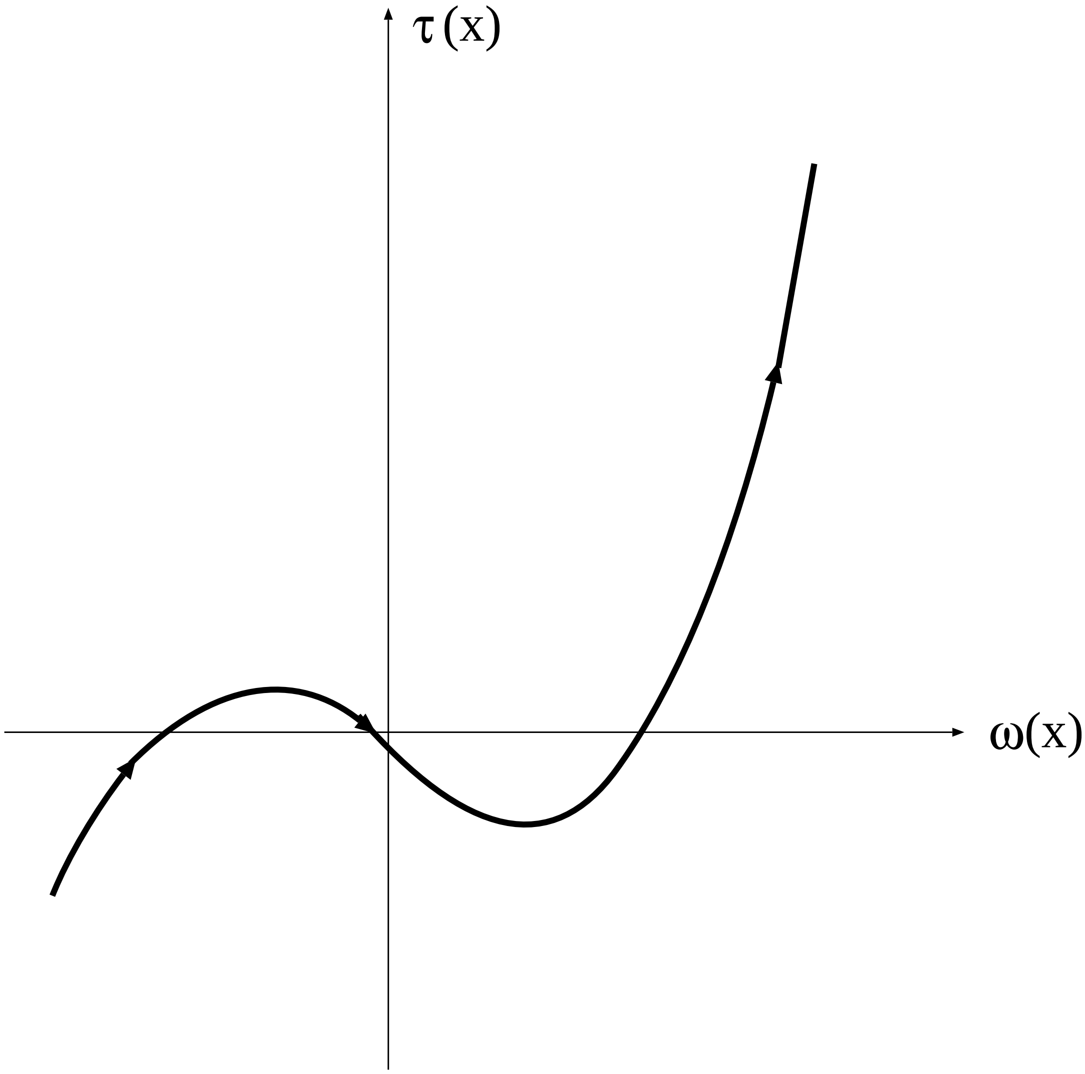

Fig. 1

Figure 1: A schematic representation of eq. (2.11) for fixed A_μ.